Exam 3 PHYS3300 Fall 2026 Name:__________________

Modern Physics Dr. Colbert

Constants:
$c=3.00\times10^8$ m/s
$m_e=0.511\mathrm{MeV}/c^2$
$m_p=938\mathrm{MeV}/c^2$
$h=6.626\times10^{-34}$J s$=4.136\times10^{-15}$ eV s
$e=1.602\times10^{-19}$C

1. Write out expressions for each of the following. (20 pts)—no numbers, constants only.

a) The energy levels of hydrogen predicted by the Bohr model (no correction for reduced mass)

$$E_n = -\frac{Z^2}{n^2}\frac{m_e e^4}{8\epsilon_0^2 h^2} = -\frac{Z^2}{n^2}(13.61\,\mathrm{eV})$$

.

b) The time independent one dimensional Schrodinger wave equation.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

c) The formula giving energy levels for an **infinite** one dimensional square well (particle of mass m and well of width L).

$$E_n = \frac{n^2 h^2}{8mL^2} \quad \text{or} \quad \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$

d) Stationary state wave functions for a particle in an infinite one dimensional well (well going from 0 to L), give possible values for "n" also?

in well $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$

$n = 1, 2, 3 \ldots$

outside $\psi = 0$

2. Given a wave function $\psi(x) = A\ [e^{-|x|-i\cos(x)}]$ valid for all x, with real value for A:

a) Set up an expression to solve for the normalization constant. JUST SET IT UP. (10 pts)

$$1 = \int_{-\infty}^{\infty} \psi^* \psi \, dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-2|x|} \, dx$$

$$A^2 = \frac{1}{\int_{-\infty}^{\infty} e^{-2|x|} \, dx}$$

can do $0 \rightarrow \infty$ & $\times 2$

b) Set up an expression to determine the probability of finding the particle between x=-0.500 and x=0.500

$$P = A^2 \int_{-0.5}^{0.5} e^{-2|x|} \, dx$$

3. In the Bohr model for simple hydrogen we originally did not consider the reduced mass. Answer the following qualitative questions regarding what happens when reduced mass is considered? (15 pts)

a) In the original model without consideration of reduced mass, the results are equivalent to using an

(<u>Infinite Proton mass</u> OR <u>Zero Proton mass</u> OR <u>Normal Proton mass</u>)

for the nucleus ---(circle one underlined answer)?

b) When reduced mass is considered, the **magnitude** of energy levels are_(________________)_ compared to the original Bohr Model without considering reduced mass of the nucleus (circle one underlined answer)

<u>Increased</u>, **<u>Decreased</u>** **<u>Remain the same</u>**

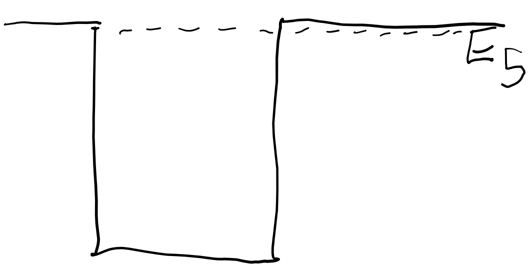
Closer to ∞ mass nuc than protium

c) Deuterium is Hydrogen but the nucleus has 1 neutron in addition to its lone proton. The transition energy for state n=3 to n=2 for Deuterium is ______________compared to the Bohr Model result for real one proton hydrogen--protium (circle one underlined word)

<u>Increased</u> **<u>Decreased</u>** **<u>Remain the same</u>**

4. You are given a particle bound in a **FINITE** square well, where there are 5 bound state solutions. The highest energy bound state is just under (at) the top of the well. (20 pts)

a) What happens to the energies and wavelength of the wave functions when the width of the well is decreased slightly? How many bound states will there be?



λ shorter

E_n increases

~~E_5 gets bumped out~~

4 states

b) What happens to the energies and wavelength of the wave functions as the height of the well potential is reduced slightly from the original?

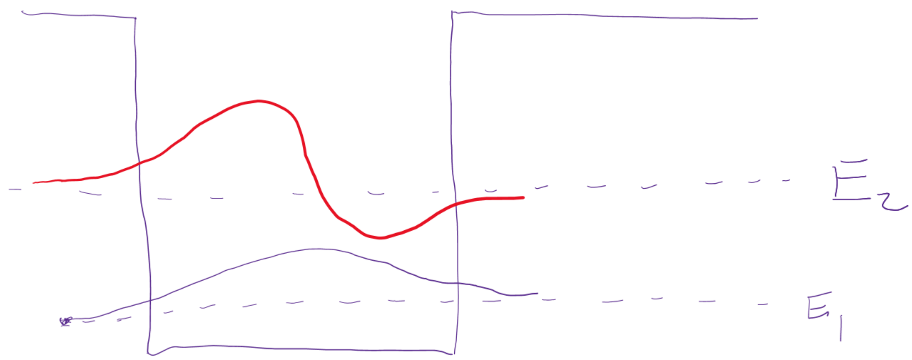
λ increases – softer walls

E_n decrease

c) For the original case how do the energies of the finite square well compare (first five bound levels) to energies found for an infinite square well with the same width?

all lower – see b

d) Give a qualitative sketch of the first two wavefunctions (two lowest energy states) for the **finite** square well. Be careful.



In each case there is some leakage of wavefunction outside the well, with wavefunction going asymptotically to zero.

5. A particle is in the **n=3 state of the infinite square well**. Set up an expression to: (20 pts)—You are given m, L and the well runs from 0 to L.

a) Write out an expression to determine the probability of the particle being between 0.200L and 0.500L. Be complete using m and L, you may use k if you specify exactly what k is. DON'T DO THE INTEGRAL.

$$P = \int_{0.2L}^{0.5L} \frac{2}{L} \sin^2\left(\frac{3\pi x}{L}\right) dx \qquad K = \frac{n\pi}{L}, \; n = 3$$

b) For the given state, set up an expression to determine the average of the quantity (AKA, expectation value) of $\sqrt{x}$ over the entire well width. SET UP—DON'T DO THE INTEGRAL.

$$\langle \sqrt{x} \rangle = \int_0^L \frac{2}{L} \sin^2 \frac{3\pi x}{L} \sqrt{x}\, dx$$

OK if use 13.60 who cares

6. Consider the energy levels of an electron "orbiting" in singly ionized lithium (so it is hydrogen-like). **Do not bother to consider reduced mass here.** The Lithium nucleus has 3 protons and 4 neutrons. (15 pts total)

a) What are the electron energies for the lowest three bound states (in eV)?

$E_n = -13.61 \frac{Z^2}{n^2}$ $\quad Z = 3 \quad n = 1, 2, 3$

$E_1 = -122.49$ eV

$E_2 = -30.62$

$E_3 = -13.61$

b) List the wavelengths for each of the transitions (in nm).

$hc = 1.2408 \times 10^{-6}$ eV m

$\lambda = \frac{hc}{\Delta E}$ $(\Delta E = h\nu)$

$\lambda_{32} = 72.9$ nm

$\lambda_{31} = 11.4$ nm

$\lambda_{21} = 13.5$ nm

c) Give the radius of highest energy level orbit for the levels in part a.

$r_n = \frac{Z^2}{n} r_0$

$= 3 \times (0.053 \text{ nm})$

$= 0.159$ nm